



**5th International Online Conference on Mathematics**  
*“An Istanbul Meeting for World Mathematicians”*  
**1-3 December 2021**

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**Editor Kenan Yildirim**

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**5th International Online Conference on Mathematics**  
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**Chair Opening Speech**

Dear Participants,

First of all, thank you very much for your interest in International Online Conference on Mathematics, 1- 3 December 2021, Istanbul, Turkey. This is the fifth edition of the our conference and we are happy due to high interest to conference.

This year, our 150 participants are from 25 different countries which are Pakistan, Algeria, Morocco, China, Greece, Kuwait, Albania, Serbia, Bulgaria, Croatia, Nigeria, India, Italy, Iran, United Kingdom, United States, United Arab Emirates, Tunisia, Azerbaijan, Uganda, Congo, Spain, Romania, Poland, Russia, Oman and France.

Also, on behalf of organizing committee, I present our deepest and special thanks to our Keynote Speakers Guiseppe Conte, Mehmet Emir Köksal and Sakthivel Rathinasamy due to their contributions to conference.

The sixth edition of this conference will be organized as face to face and online in the end of the June in Istanbul. I hope that you will enjoy. Thank you very much for your participation and interest.

Kenan Yıldırım, PhD

Chair on ICOM'21

01 December 2021



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**$p$  – valently Convex of Complex Order for a General Integral Operator**

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**Abstract**

In this study, we defined a new general  $p$  – valent integral operator in the unit disk  $\mathcal{U}$ . We obtained some sufficient condition for the integral operator  $\mathcal{F}_{p,m,l,\mu}(z)$  to be  $p$  – valently convex of complex order.

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**A Block Hybrid Scheme for the Solution of First Order Ordinary Differential Equations**

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**Abstract**

A new hybrid linear multistep method (LMM) is considered for the solution of first order initial value problems (IVPs). The new hybrid method is an extension of LMM by the inclusion of extra intermediate off-step points in between the usual grid points in the numerical schemes. A detailed analysis of the method such as the local truncation error and order, consistency and zero stability are investigated and presented. The method as compared with other recently derived methods provide approximation of high accuracy to solution of IVPs in ordinary differential equations.

**Keywords:** Hybrid linear multistep method, Off-step points, Local truncation error, Consistency, First order initial value problems, Zero-stable.

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**A Crowd Dynamic-based Approach to Calculate Contact Matrices for Epidemiological Models**

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**Abstract**

Social contact matrices represent a crucial tool to assess the reproduction rate of disease transmission in epidemiological models. Projection methods are usually used to determine the contact matrices in countries where conducting surveys is not easy [1, 2]. Furthermore, these obtained matrices using projection and survey methods do not consider the impact of social distancing and non-pharmaceutical interventions on the behavior of individuals. In this work, we estimate the contact matrices in different locations (malls, households, workplaces, and schools), using a microscopic social-force model that considers the population demographics and location-specific movement characteristics. The proposed approach allows for the proper integration of social distancing and NPIs into compartment-based epidemiological models.

**References:**

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**A Hybrid Genetic Algorithm for Improving the Approximate Solutions for Travelling Salesman Problem**

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**Abstract**

There are some approximation algorithms for travelling salesman problem, whose approximation ratios are tight. So, it is a challenge to transform the obtained approximate solution into a nearly optimal solution. The genetic algorithm is an evolutionary algorithm and 2-opt heuristic method is a local search algorithm; then, a hybrid algorithm is developed to improve an approximate solution and to avoid from such a local optimal solution. The genetic algorithm does not yield high quality population for the initial approximate solution, because of local optimality; 2-opt heuristic as well needs many iterations to obtain a good solution. The Christofides algorithm provides a 1.5-approximate solution for the symmetric travelling salesman problem, where the arc costs satisfy the triangle inequality. So, the approximate solution is applied to produce the initial population for the genetic algorithm. Then, the crossover and the mutation operators produce some offspring by 2-opt heuristic method. Therefore, the hybrid algorithm could produce a near optimal solution in a reasonable iteration. It is implemented on some travelling salesman instances, and the results determine the hybrid algorithm efficiency.

**References:**

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**A Mathematical Model of Hysteresis in Soft Magnetic Materials**

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**Abstract**

In this paper it is presented a mathematical approach of the magnetic hysteresis in soft magnetic materials (SMMs) using an efficient classical Preisach model. The magnetic behavior of SMMs is characterized by a function  $p(a, b) = p_1(a)p_2(b)$ , which represents the density distribution of the rectangular hysteresis loops of different reversal fields  $a$  and  $b$ , ( $b < a$ ,  $a \leq H_s$ ,  $b \geq -H_s$ ), where  $H_s$  is the saturation magnetic field. The experimental data of the normal magnetization curve must have  $N$  points equally spaced, obtained by varying the applied magnetic field  $H$  from zero to saturation value  $H_s$ . From  $H_s$  value is necessary to have an equally spaced distribution of  $N$  points to zero value and another  $N$  points from zero to negative saturation value  $-H_s$ . Because of the symmetry of the Preisach distribution the function  $p_1$  is defined by  $N$  values and  $p_2$  is characterized by  $2N$  values. The entire Preisach distribution  $p$  will have  $3N$  equations. The knowledge of the function  $p(a,b)$  is sufficient to determine the macroscopic behavior of the material by computing the magnetic polarization  $J$ . The Preisach density  $p(a,b)$  exists, is unique and it depends only on the nature and structure of the material. The set of the  $3N$  equations can be described as follows, for  $0 \leq k \leq N - 1$ :

$$J_{k+1} - J_k = \frac{H_s^2}{N^2} p_1(a_{k+1}) \sum_{i=0}^k [p_2(b_{N-i}) + p_2(b_{N+i+1})];$$

$$J_{N+k} - J_{N+k+1} = \frac{H_s^2}{N^2} p_2(b_{k+1}) \sum_{i=0}^k p_1(a_{N-i});$$

$$J_{2N+k} - J_{2N+k+1} = \frac{H_s^2}{N^2} p_1(a_{k+1}) \left[ \sum_{i=0}^k [p_2(b_{N-i}) + p_2(b_{N+i+1})] + \frac{H_s^2}{N^2} p_2(b_{N+k+1}) \sum_{\substack{i=k+1 \\ i \leq N}}^N p_1(a_i) \right];$$

$$J = \iint_{S_+} p(a,b) da db - \iint_{S_-} p(a,b) da db,$$

**Acknowledgement:** The work was supported by University Politehnica of Bucharest through the “Engineer in Europe” project of the Romanian Ministry of Education and Research, project number 140/GP/19.04.2021.

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**A new approach for computing the inverse of confluent Vandermonde matrices via Taylor's expansion**

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**Abstract**

The importance of Vandermonde matrices and their inverses hardly need to be explained. They have realized serious importance, due to their normal appearance in diverse areas of mathematics, physics, and engineering. Both the Vandermonde matrices and their inverses have been widely used in many applications including interpolation, discrete Fourier transform, coding theory, hypersurfaces, systems theory, etc. Many mathematicians were involved in the study of the Vandermonde matrices. D. Kalman shows in [1] that Vandermonde matrices have realized serious importance, due to their normal appearance, in differential and difference equations. In [2], we can see the importance of the usual Vandermonde matrix in cryptography, it is a helpful tool in decoding the Reed-Solomon codes. Also, we can see some applications of Vandermonde matrices in V-independent sets in [4]. The inverse of the Vandermonde matrices has received a regain of interests owing to its diverse applications. Therefore, several methods and algorithms have been developed to achieve such inversion. Explicit formulas for the inverse of the Vandermonde matrices are given in [5],[7],[8],[6],[9],[5],[10],[11]. There are two famous methods of calculating the explicit inverse of the usual Vandermonde matrix. The first one is the LU factorization (see [11] and [12]). The second inversion method is based on the use of the elementary symmetric functions of roots (see [7],[9],[10]). The partial fraction expansion is the most widely used method for computing the elements of the inverse of the confluent Vandermonde matrix (see for example [3]). This talk aims to expose our novel method for computing an explicit formula for the inverse of the confluent Vandermonde matrices. There is a lack of explicit formulas expressing their inverses, which are desired by the mathematical community. Our results in a large part fill that hole. The presented formulas are compact and easy to use, and quite innovative. It is well known that obtaining the inverse of this kind of matrices is fairly difficult but with our method, we reduce the problem to a simple problem of finding the standard form of certain polynomials. The advantage of the proposed method is that it is direct and straightforward and gives explicit and compact formulas not current in the large literature on this essential subject. The proposed results may have many interesting perspectives in diverse areas of mathematics and natural sciences, notably on the situations where the Vandermonde matrices have acquired much usefulness. The main tools are some elementary basic linear algebra.

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**A new approach for solving distributed order fractional partial differential equations**

Monireh Nosrati Sahlan

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**Abstract**

Distributed-order fractional partial differential equations (DOFPDEs), can serve as a natural generalization of the single order and multi-term fractional partial differential equations. Distributed-order fractional derivatives indicate fractional derivatives that are integrated over the order of the differentiation within a given range. In the current paper, we consider the following two-dimensional DOFPDE

$$\int_0^1 \rho(\alpha) D_c^\alpha u(x, t) d\alpha = u_{xx}(x, t) + H(x, t), \quad (x, t) \in [0, 1] \times [0, 1], \quad (1)$$

subject to the initial - Dirichlet conditions:

$$u(x, 0) = f(x), \quad u(0, t) = p(t), \quad u(1, t) = q(t), \quad (2)$$

where  $D_c^\alpha u$  denotes the Caputo fractional derivative and  $H, f, p$  and  $q$  are continuous functions. Also  $\rho(\alpha)$  is a continuous non-negative weight function satisfying

$$\rho(\alpha) \geq 0, \quad \int_0^1 \rho(\alpha) d\alpha > 0.$$

Also the fractional order derivative,  $\alpha$ , is considered in the Caputo sense. The aim of this study is introducing a new approach based on fractional order Bernoulli wavelets. The operational matrices of Bernoulli wavelets and the Gauss-Legendre quadrature are applied to acquire the approximated solution of problem (1)-(2). To validate the proposed method, we have considered some illustrative examples and compared with the exact results.

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**A New Type of Dual Numbers**

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**Abstract**

In this study, we introduce new type of dual numbers by means of Leonardo numbers. We call them “dual Leonardo numbers”. We establish the properties of dual Leonardo numbers including relations with Leonardo, Fibonacci and Lucas numbers. The identities D’ocagnes, Cassini, Catalan and dual numbers with negative index are given. Additionally, we obtain Binet formula and generating function for dual Leonardo numbers. Finally, we exemplify the theorems.

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**A Note on the Comparison Theorems for Second Order Neutral**  
**Dynamic Equations on Time Scales**

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**Abstract**

In [1], Agarwal et al. presented some comparison theorems on the oscillation of second order functional dynamic equations with a neutral term. They studied a class of neutral dynamic equations under assumptions that allow applications to equations with both delayed and advanced arguments. In this work, by extending the ideas exploited in [1] and [2], we attempt to establish several new comparison theorems for oscillation of second order mixed neutral noncanonical dynamic equations, based on comparisons with associated second order linear non-neutral dynamic equations on time scales.

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**A special Green-function for steady-state fractional differential equations with nonlocal boundary condition**

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**Abstract**

In this work, we consider steady-state fractional convection-diffusion equation with nonlocal boundary condition. Here, the highest-order derivative is taken as Caputo fractional derivative with order of  $1 < \gamma < 2$ . An explicit formula for the associated Green's function is given in terms of two-parameter Mittag-Leffler functions.

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**A Study of Stability of Differential Equations in the Sense of Ulam**

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**Abstract**

In this seminar study, Ulam Type Stability Of Differential Equations was studied. Here, we examined how stability started, how it evolved, and the methods developed. We searched the literature and made a compilation of it. First of all, information was given about Ulam's life and the stability problem he presented. Afterwards, the answers of Hyers and Rassias to this stability problem are examined. Then, some of the studies from the stability of the simplest differential equations to the stability of the delayed differential equations are given.

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**A Study of the Solutions of Notable Engineering Problems**

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**Abstract**

We develop some fresh findings involving graph contractions in a more generalized environment. We also provide some examples to further illustrate and expound on the applicability of the obtained findings. We explain how our conclusions enhance, extend, generalize, and unify various notable results in the existing state-of-the-art using nontrivial cases. We use computer simulation to verify our findings. To pique interest in the topic and demonstrate its utility, we focus on recent applications, placing emphasis on the existence of solutions to various models related to engineering problems, such as fourth-order two-point boundary value problems describing deformations of an elastic beam, ascending motion of a rocket, and a class of integral equations. This is a novel method that will lead to new directions in the underlying graph structure.

**References:**

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**Algorithm For Solving A Transportation Problem With Five Indicators**

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**Abstract**

In this paper, I studied an algorithm for obtaining the best solution to transportation problem a fixed-charge with five indicators (AATPI5FC) .The introduced method solves FCTPI5 to analyze the problem into partial subsections. Solution algorithm is coupling between our technique and simplex algo-Rithm (the technique on the fifth index to guarantee improved service).

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**An Analytical Approach to Assess and Compare the Vulnerability Risk of Operating Systems**

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**Abstract**

Operating system (OS) security is a key component of computer security. Assessing and improving OSs strength to resist against vulnerabilities and attacks is a mandatory requirement given the rate of new vulnerabilities discovered and attacks occurring. Frequency and the number of different kinds of vulnerabilities found in an OS can be considered an index of its information security level. In the present study five mostly used OSs, Microsoft Windows (windows 7, windows 8 and windows 10), Apple's Mac and Linux are assessed for their discovered vulnerabilities and the risk associated with each. Each discovered and reported vulnerability has an Exploitability score assigned in CVSS score of the national vulnerability database. In this study the risk from vulnerabilities in each of the five Operating Systems is compared. Risk Indexes used are developed based on the Markov model to evaluate the risk of each vulnerability. Statistical methodology and underlying mathematical approach is described. Initially, parametric procedures are conducted and measured. There were, however, violations of some statistical assumptions observed. Therefore the need for non-parametric approaches was recognized. 6838 vulnerabilities recorded were considered in the analysis.

According to the risk associated with all the vulnerabilities considered, it was found that there is a statistically significant difference among average risk levels for some operating systems, indicating that according to our method some operating systems have been more risk vulnerable than others given the assumptions and limitations. Relevant test results revealing a statistically significant difference in the Risk levels of different OSs are presented.

**Keywords:** Cybersecurity, Markov chain, Non-parametric analysis, Vulnerability.



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**Analysis of a cancer-obesity model under the treatment with quadratic optimal control approach**

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**Abstract**

In this paper, the author proposes a cancer-obesity-treatment model involving four ordinary differential equations under some control parameters, including IL-2 therapy, ACI therapy, and nutritional diet. The author begins the work by examining the proposed model without applying any treatments. Then the existence and stability of the proposed model at each of equilibrium under the treatment procedure are investigated. Next, an optimal control problem is constructed over a finite time interval under the applied treatment control parameters to reduce the number of cancer cells and to minimize the toxicity effect of the applied drug dose on other healthy cells. Finally, the author presents simulation results for the proposed model considering different treatment strategies viz. no treatment, only IL-2 therapy, a combination of IL-2 therapy and ACI therapy, and a combination of IL-2 therapy, ACI therapy, and diet to understand the effect of treatment. The findings shows that it may be possible to create an optimal treatment schedule for cancer management by controlling all three treatment parameters.

**Keywords:** cancer-obesity model; adoptive cellular immunotherapy (ACI-therapy); IL-2 therapy; nutritional diet; stability; optimal control.

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**Analyzing the performance of French and German car brands by using  
a new simultaneous analysis method**

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**Abstract**

We analyze the data of French car brands which constitute the first multi-table and the German car brands, the second multi-table, by using a dual data analysis method: CONCORGS1D. This method is an extension of CONCORGS1 method in the context of two horizontal multi-tables measured on the same set of individuals. Each brand is made up of several models and these car models from two vertical multi-tables measured on the same number of variables.

**Keywords:** CONCORGS, Dual method, Horizontal multi-table, Vertical multi-table

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**Application of Fractional Calculus Operators to the functions in the certain subclasses of analytic functions**

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**Abstract**

In this study, we investigate the growth and distortion properties of functions in the a certain subclasses of analytic functions which involves the operator Fractional Calculus.

**References:**

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**APPROXIMATE CONTROLLABILITY OF NONSIMPLE ELASTIC PLATE WITH MEMORY**

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**Abstract**

In this paper, we give some qualitative results on the behavior of a nonsimple elastic plate with memory corresponding to anti-plane shear deformations. First we describe briefly the equations of the considered plate and then we study the well-posedness of the resulting problem. Secondly, we perform the spectral analysis, in particular, we establish conditions on the physical constants of the plate to guarantee the simplicity of the roots of the characteristic equation. The spectral results are used to prove the exponential stability of the solutions at an optimal decay rate given by the physical constants. Then we present an approximate controllability result of the considered control problem. Finally, we give some numerical experiments based on the spectral method developed for multi-dimensional problems with implementation in MATLAB for one and two-dimensional problems.

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**Approximation by Nonlinear Multivariate Convolution Operators in Differentiation Sense**

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**Abstract**

In this work, we will prove some theorems for nonlinear multivariate convolution operators in order to approximate one-sided partial derivatives of functions of multivariables by using extended definition of the notion of one-sided derivative in univariate case. Our proofs will be guided by previous studies in the literature.

**Keywords:** nonlinear multivariate convolution operators, Lipschitz condition, approximation of partial derivatives

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**Approximation of stochastic differential equations with jumps under irregular coefficients**

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**Abstract**

In this talk, we consider a stochastic differential equation with jumps for which pathwise uniqueness hold. We establish a fundamental mean square convergence theorem for Euler approximation scheme. We provide some results on strong stability with respect to small perturbations on the initial conditions, and study the convergence of Picard scheme.

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**Artificial neural networks for non-parametric regression with biological data**

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**Abstract**

Deep neural networks have huge success in different areas such as image progressing, analyses of biomedical signals and financial time series in recent years. In this work, we have suggested this neural network model as generalized additive models (GAMs) for biological data. Because, GAMs have great flexibility to explain explanatory variables while capturing non-linearities in the regression model. In order to solve the problem of non-linearity, Hastie and Tibshirani (1990) [3] have suggested the multilayer feed forward neural network approach. Then, the non-parametric regressions are proposed by Schmidt-Hieber (2020) [4], in particular, when the number of network's parameter is greater than the number of samples which is called the problem of over-parametrization. Moreover, we have also compared the proposed generalized additive neural networks (GANNs) and Gaussian Graphical model (GGM) with bootstrap scheme and two model selection criteria inserted in GGM. we consider the deep learning network model in the construction of biological networks. This approach has been commonly used in recent years for modelling complex non-linear regression model and for the classification problem. Here, we have adapted this approach as a regression model for the representation of the protein-protein interaction networks. Thus, we have compared its performance with the Gaussian graphical model (GGM) which is one of the well known graphical models to describe the biological systems. In the calculation of GGM, we have also implemented the bootstrap procedure to increase the number of observations and the Consistent AIC as well as information and complexity approaches as the model selection criteria within GGM to improve the accuracy of estimates. In the analyses, we have used two real bench-mark datasets and compared the accuracy of the deep learning method with the underlying GGM.

**Keywords:** Deep neural networks, non-parametric regression, biological data

**References:**

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**Bivariate Max-Product Bleimann–Butzer–Hahn Operators**

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**Abstract**

In the approximation theory, polynomials are particularly positive linear operators. Nonlinear positive operators by means of maximum and product were introduced by B. Bede. In this study, nonlinear maximum product type Bivariate Bleimann–Butzer–Hahn Operators are defined and approximation properties are investigated with the help new definitions.

**References:**

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**Coefficient Estimates For A Certain Subclass of Bi-Univalent Functions Defined By using Deniz-Özkan Differential Operator**

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**Abstract**

In this paper, we investigate a new subclass  $B_{\Sigma}^m(\lambda, \beta; \varphi)$  of bi-univalent functions in the open unit disk  $U$  defined by Deniz-Özkan differential operator. We obtain initial coefficients bounds for functions belonging to this class.

**References:**

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**Cofinitely  $\oplus$ -g-Rad-Supplemented Modules**

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**Abstract**

In this work, all rings have unity and all modules are unitary left modules. Let  $M$  be an  $R$ -module. If every cofinite submodule of  $M$  has a  $g$ -supplement that is a direct summand in  $M$ , then  $M$  is called a cofinitely  $\oplus$ -g-radical supplemented (briefly, cofinitely  $\oplus$ -g-Rad-supplemented) module. In this work, some properties of these modules are investigated.

**Keywords:** Small Submodules,  $g$ -Small Submodules, Supplemented Modules,  $g$ -Supplemented Modules.

**Results**

**Proposition 1.** Every  $\oplus$ -g-Rad-supplemented module is cofinitely  $\oplus$ -g-Rad-supplemented.

**Proposition 2.** Every finitely generated cofinitely  $\oplus$ -g-Rad-supplemented module is  $\oplus$ -g-Rad-supplemented.

**Proposition 3.** Every cofinitely  $\oplus$ -g-supplemented module is cofinitely  $\oplus$ -g-Rad-supplemented.

**Proposition 4.** Every  $\oplus$ -cofinitely supplemented module is cofinitely  $\oplus$ -g-Rad-supplemented.

**Proposition 5.** Hollow and local modules are cofinitely  $\oplus$ -g-Rad-supplemented.

**References:**

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**Coincidence and Common Fixed Soft Point Theorems in Parametric Soft Metric Spaces**

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**Abstract**

Parameters play the main role to model real life problems involving uncertainly. Researchers working in the area of metric spaces have been inspired by this idea, and the soft metric spaces gave birth [1]. By the similar reason, we defined parametric soft metric spaces as the parametric extension of the soft metric spaces [2]. The existence and the uniqueness of the fixed points in the metric (-like) spaces take important place. Since the theory of fixed points is the backbone of the several applied sciences. In this article, we investigate some common and coincidence fixed soft point results in the parametric soft metric spaces. So we hope to make contribution to the fixed point theory.

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**Commutativity and Wangerin Differential Equation**

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**Abstract**

The commutativity conditions of the Wangerin differential equation are studied. It is shown that the only class of commutative pairs of Wangerin differential equation is its constant feedback and constant-forward feedback conjugate pairs, and the possibility of commutative pairs other than feedback conjugates is proven to be strictly negative. The commutativity with its feedback conjugate pairs is also studied under nonzero initial conditions.

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**Commutativity Associated with Euler Second-Order Differential Equation**

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**Abstract**

This work study the commutativity and alongside with the sensitivity of second-order Euler differential equation. The conditions for commutativity of second-order Euler differential equation are investigated. Example will be given to support the results.

**Keywords:** Commutativity, Euler Differential Equation and analogue system

**References:**

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**Commutativity of a Third-Order Discrete-Time Linear Time-Varying System**

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**Abstract**

In this study, necessary and sufficient conditions for the commutativity of third-order discrete-time linear time-varying systems are derived. The problem is formulated for third-order difference equations. The results are well verified by examples worked by using MATLAB Simulink tool. The importance of the paper subject is emphasized considering the engineering applications such as sensitivity, robustness and stability.

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**Comparison of Static Path Planning Models by Time Requirement**

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**Abstract**

In the last decade, much research has been done on localization for wireless sensor networks. Most existing localization techniques for WSNs can be classified into two main groups based on a key classification: range-based or range-free. Range-free techniques use only connectivity information between sensors and beacons [1]. Range-based techniques use distance/angle information to locate a node, while range-free techniques use connectivity information to locate it. Range-free techniques are less complex, inexpensive, and do not require additional hardware, but range-based techniques provide better results in terms of localization accuracy [2]. The location information of a sensor node is the main problem in processing the sensed data in wireless sensor networks (WSNs). A promising solution for static distributed sensors is localization using mobile beacons. The main challenge is to design and develop an optimal path planning mechanism for a mobile beacon to reduce the time required for location determination. In this paper, we have tested six existing path plans in the literature with a different number of nodes in Uniform, Beta, Weibull, Gamma, and Generalized Pareto distributed networks. The time taken for localization of these models is compared using Accuracy Priority Trilateration (APT) localization technique. The localization performance of the networks is evaluated and compared using MATLAB simulations.

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**Comparison of the order-type integrals in Riesz Space**

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**Abstract**

In this paper we begin with investigating the order-type Pettis, Bochner, Dunford and McShane integrals in Banach lattice integrals and give some comparison results. One interesting difference between these kinds of integration is the fact that they possess the properties represented by Hake and Henstock lemmas. We observe that on the case of L-space the order integral of Pettis is stronger as Bochner one (by norm).

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**Composition series of a class of induced representations**

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**Abstract**

We determine composition series of a class of a parabolically induced representation of  $p$ -adic symplectic group in terms of Moeglin-Tadić classification of discrete series. We consider representations induced from two essentially square integrable representation attached to Zelevinsky segments, with certain half-integer borders, and a cuspidal representation of a smaller symplectic group.

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**Convergence of an implicit scheme for diagonal non-conservative hyperbolic systems**

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**Abstract**

In this work we present a convergence result for an implicit Upwind scheme considering the framework of hyperbolic systems, which are not necessarily strictly hyperbolic [2]. Related to this work, it is worth noting that, in [3] the authors have proved a similar result for a semi-explicit scheme in the case of non-conservative strictly hyperbolic systems. Moreover, their result was only valid in the class of vanishing viscosity solutions, introduced by Bianchini and Bressan in [1]. Here, we show the convergence taking only Lipschitz continuous solutions, without any other restriction concerning the class of solutions.

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**Curvatures of the Astro-Rotational Hypersurfaces**

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**Abstract**

In this work, we introduce and examine differential geometric properties of the astro-rotational hypersurface which its profile curve has astroid curve in the four dimensional Euclidean space  $\mathbb{E}^4$ . We reveal the curvatures  $\mathfrak{C}_{i=1,2,3}$  of the astro-rotational hypersurface, that is the first (ie. the mean) curvature  $\mathfrak{C}_1$ , the second curvature  $\mathfrak{C}_2$ , the third (ie. the Gaussian) curvature  $\mathfrak{C}_3$ . Moreover, projecting the the astro-rotational hypersurface into 3-spaces, we indicate surfaces, visually.

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**Cut locus of  $L^1$  sub-Finsler problems in  $\mathbb{R}^3$ : two case studies.**

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**Abstract**

Let  $f$  and  $g$  be two smooth vector fields on  $\mathbb{R}^3$  such that  $f, g$  and their Lie brackets  $[f, g]$  are linearly independent at every point. We endow  $\mathbb{R}^3$  with a  $L^1$  sub-Finsler distance, in the following way: for every pair of points  $x_0, x_1$  in  $\mathbb{R}^3$ , we define the distance between them as the infimum of the functional

$$J(u) = \int_0^1 |u_1(t)| + |u_2(t)| dt,$$

taken over all  $L^1$  functions  $u_1$  and  $u_2$  such that the solution of the Cauchy problem

$$\begin{cases} \dot{\xi}(t) = u_1(t)f(\xi(t)) + u_2(t)g(\xi(t)) \\ \xi(0) = x_0 \end{cases} \text{ satisfies } \xi(1) = x_1.$$

This problem is indeed an optimal control problem, that can be treated with classical tools, such as Pontryagin Maximum Principle, which provides a family of candidate optimal trajectories, called *geodesics*. In most cases, a geodesic is not optimal forever, but at some points, called the *cut point*, it loses its optimality (that is, it ceases to be length minimizing). The set of all cut points is called the *cut locus*. Determining the cut locus is thus important in order to compute the distance between two points and to describe the properties of the metric, such as, for instance, the shape of the spheres. In the nilpotent case (the Heisenberg group), studied in 1., the cut locus is made of four symmetric branches. The generic case is studied in paper 2.: in particular, depending on the values of two invariants (assumed to be non-zero) associated with the fields  $f$  and  $g$ , the cut locus is symmetric with respect to the origin, and is composed either by five smooth branches, or by only one smooth branch. When one (and only one) of the invariants is nonzero, there are several possible cases, some of them being quite different from the generic ones (for instance, the central symmetry is broken): in particular, in this talk we are showing two of them, one in which the cut locus is made by three smooth branches, one other in which the cut locus is not connected.

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**Development of a neural statistical model for the relative humidity levels prediction in the region of  
Rabat-Kenitra (Morocco)**

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**Abstract**

This work provides the development of a powerful artificial neural network (ANN) model, for the prediction of relative humidity levels, using other meteorological parameters of the Rabat-Kenitra region. The treatment was applied to a database containing a daily history of five meteorological parameters of nine stations covering this region for a period from 1979 to mid-2014.

We have shown that for the prediction of relative humidity in this region, the best performing three-layer ANN (input, hidden and output) mathematical model is the multi-layer perceptron (MLP) model. This neural model using the Levenberg-Marquard algorithm, having an architecture [5-11-1] and the transfer functions Tansig in the hidden layer and Purelin in the output layer was able to estimate values for relative humidity very close to those observed. Indeed, this was affirmed by a low mean squared error (MSE) and a high correlation coefficient (R), compared to the statistical indicators relating to the other models developed as part of this study.

**Key words** — Modeling; ANN; MLP, learning algorithm; Relative humidity; Rabat-Kenitra.

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**Discretization of Continuous Random Distributions Based on the Minimization of a Statistical Distance between Cumulative Distribution Functions**

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**Abstract**

Approximating a continuous random variable through a discrete one can be a possible solution to be adopted in many problems where dealing with continuous probability distribution would make computations intractable or would dramatically increase the computational burden required to produce a numerical solution [1]. In this work, we propose a technique for constructing a discrete approximation of a continuous probability distribution, which is based on the minimization of a statistical distance between the original random distribution and its discrete approximation, which is expressed in terms of the two cumulative distribution functions and which can be selected from a broad class containing among others the Cramér-von Mises distance. The proposed technique is able to handle both finite and countable supports for the discrete approximation; in the former case the support points and their probabilities are jointly derived from the minimization process; in the latter case, the support points are fixed a priori and they constitute the set of integers or non-negative integers according to whether the original random variable is supported over the whole or positive real line. When the support points are fixed, nice closed-form expressions are available for the probability mass function of the discrete approximation for most parametric families of continuous distributions. An application to a well-known problem in the insurance field is provided.

**Keywords:** Anderson-Darling distance, Cramér-von Mises distance, discrete approximation

**Acknowledgement:** This project is supported by Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA)

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**Doubly Reflected Backward Stochastic Differential Equations in the Predictable Setting**

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**Abstract**

In this paper, we introduce a specific kind of doubly reflected backward stochastic differential equations (in short DRBSDEs), defined on probability spaces equipped with general filtration that is essentially non quasi-left continuous, where the barriers are assumed to be predictable processes. We call these equations predictable DRBSDEs. Under a general type of Mokobodzki's condition, we show the existence of the solution (in consideration of the driver's nature) through a Picard iteration method and a Banach fixed point theorem. By using an appropriate generalization of Itô's formula due to Gal'chouk and Lenglart we provide a suitable a priori estimates which immediately implies the uniqueness of the solution.

**Keywords** Doubly reflected backward stochastic differential equations · Predictable DRBSDEs · Non-quasi-left continuous · Picard iteration method · Fixed point theorem



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**Dynamics of a Biological System with Discontinuous Effects**

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**Abstract**

The theory of differential equations provides very important tools to understand and interpret many real world processes in several fields. This theory has been developed significantly and extended to different types of differential equations, such as differential equations with piecewise constant argument, due to the needs. This paper is devoted to a prey-predator system modeled by differential equations with piecewise constant argument of generalized type [Akhmet, 2007]. The importance of these equations is caused by the needs of modern science and technology as discontinuous characteristics are very often observed in the growth of real processes in biology, chemistry, control theory, ecology, economics, neural networks, medicine, electronics, physics, and another field.

The main objective of this paper is to analyze the dynamics of a prey and predator system [Xu and Li, 2012] with piecewise constant argument of generalized type. We will investigate and present sufficient results on positive invariance for this system [Akhmet, Aruğaslan and Liu, 2008].

**Acknowledgement:** Differential Equations with Piecewise Constant Argument, Positive Invariance, Prey-Predator Model, Functional Response, Nonautonomous System.

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**Eigenvalue estimates for magnetic Schrodinger operators in a waveguide**

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**Abstract**

The study of quantum waveguides has attracted much interest in the recent years largely because of their physical importance. They represent many important applications in nano physical devices as well as flat electromagnetic waveguides. Several interesting results on the spectral properties of these quantities have been obtained, including results on the existence of eigenvalues below the essential spectrum. Some of these results largely depend on the geometry of the waveguide and the conditions imposed at the boundary. In the present work, we present an upper estimate for the number of negative eigenvalues below the essential spectrum for the magnetic Schrödinger operator with Aharonov-Bohm magnetic field in a strip. The estimate is obtained by reducing the operator to a family of self-adjoint Sturm Liouville operators whose estimates for the number of negative eigenvalues below the essential spectrum are given by the well-known Bergman type estimates

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**Energy of Timelike spherical Magnetic Curves on the De-Sitter Space  $S_1^2$**

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**Abstract**

In this paper, we investigate the energy of timelike spherical magnetic curves associated with the given magnetic field  $G$  on the De-Sitter 2-space  $S_1^2$ . We use a completely geometrical approach for this computation such that the energy of each timelike spherical magnetic curve is stated by using the geodesic curvature of each magnetic curve.

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**Energy on the  $N_f$ -Magnetic curves**

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**Abstract**

In this paper, we define a normal magnetic curve geometrically, ( $N_f$ -magnetic curve) which is associated with the magnetic field  $B$  on the 3D Riemannian manifold, by considering a normal force on the particle. Moreover, we obtain energy on the  $N_f$ -magnetic curves in the  $B$  magnetic vector field.

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**Existence of Non-Trivial Solutions For The p&q-Laplacian Problem With Discontinuous Nonlinearity**

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**Abstract**

Using non-smooth mountain pass theorem for nondifferentiable functionals, we prove the existence of non-trivial solution to a class of p-q elliptic problems with discontinuous nonlinearity in smooth bounded domain.

**Keywords:** Variational methods, Discontinuous nonlinearity, p-q Laplacian

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**Exponential synchronization of retarded BAM neural network systems**

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**Abstract**

The purpose of this work is to address the stabilization of some bidirectional associative memory neural network systems involving distributed delays. Under a relaxed Lipschitz condition, Cid criterion, and a nonlinear version of Halanay's inequality, a new sufficient condition is derived ensuring the exponential synchronization of the master-slave systems. The effectiveness of the obtained theoretical results is validated by a numerical example.

**Keywords:** Exponential synchronization, bidirectional associative memory, distributed delay, Halanay inequality.

**Acknowledgement:** The second author would like to thank King Fahd University of Petroleum and Minerals for its continuous support through project SB201006.

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**Fault-tolerant tracking control design for fractional-order nonlinear control systems**

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**Abstract**

In this talk, a fault-tolerant control that governs the tracking of fractional-order nonlinear uncertain systems with actuator faults, uncertainties and external disturbances will be discussed. By employing Lyapunov technique and theories of fractional calculus, a set of sufficient conditions is derived in the form linear matrix inequalities to design the dynamic error feedback fault-tolerant controller which guarantees the robust asymptotic stabilization of the resulting closed-loop fractional-order system. Finally, two numerical examples with simulations are provided to demonstrate the effectiveness of the obtained control design technique.

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**Feedback Control for Attractor Dynamics in the Epileptor Model**

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**Abstract**

The variety of dynamic mechanisms leading to epileptic behavior in the human brain demands the development of different theoretical approaches to the modeling of this disease. The Epileptor [1] is a generic phenomenological model recently developed to describe fast-slow limit cycles in the dynamics of seizures. To make the control over the co-existing attractors in the Epileptor model we propose here the algorithm based on Kolesnikov's "synergetic" target attractor feedback [2]. We investigate the pros and cons of our approach to compare with other methods and discuss the perspectives of the further development of the control algorithm for modeling the real epileptic processes in vivo.

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**Fekete-Szegő Problem Functional Problems For Some Subclasses of Bi-Univalent Functions**  
**Defined By Deniz-Özkan Differential Operator**

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**Abstract**

In this study, we solve Fekete-Szegő problem for a new subclass  $B_{\Sigma}^m(\lambda, \beta; \varphi)$  of bi-univalent functions in the open unit disk  $U$  defined by Deniz-Özkan differential operator.

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**Fixed point results for  $(\alpha, \mu, \varphi)$ -generalized Meir-Keeler contraction on quasi 2-normed spaces**

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**Abstract**

The normed spaces are generalized to 2-normed spaces by Gahler [1]. Some topological properties and fixed point theorems are studied by many authors in these spaces. Later, 2-normed spaces are extended to quasi 2-normed spaces by Park [2]. In 1969, Meir-Keeler[3] introduced a new contraction and proved the existence of a fixed point in metric spaces. Samet et al. [4] presented the concept of  $\alpha$ -admissible mappings and generalized many known contractions. In this paper, there are proved some results which guarantee the existence and the uniqueness of a fixed point for generalized Meir-Keeler contraction, using  $\alpha$ -admissible and  $\mu$ -subadmissible mappings using a comparison function in quasi 2-normed spaces. The main theorem generalizes some known results.

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**Focal Curves of Adjoint Curves**

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**Abstract**

In this study, we construct focal curves of adjoint curves. Moreover, we obtain new characterizations of focal curvatures.

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**Fractional Bernoulli wavelets for solving fractional Burger’s Equation**

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**Abstract**

Burgers equation is one of the basic and important non-linear partial differential equation including diffusive effects and non-linear propagation effects. Fractional Burgers equation can describe the process of unidirectional propagation of weakly nonlinear acoustic waves through a pipe filled with gas. In this study, the fractional order Bernoulli wavelets are adopted to acquire the approximate solution of one dimensional time-fractional Burger’s equation. Burgers’ equation is the diffusive equation [1]:

$$\frac{\partial^\alpha u}{\partial t} + u \frac{\partial u}{\partial x} - \nu \frac{\partial^2 u}{\partial x^2} = H(x, t), \quad (x, t) \in [0, 1] \times [0, T], \quad (1)$$

subject to the following initial and boundary conditions

$$u(x, 0) = f(x), \quad u(0, t) = p(t), \quad u(1, t) = q(t). \quad (2)$$

where  $\nu > 0$  denotes the coefficient of kinematic viscosity and the prescribed function  $f(x)$  is sufficiently smooth. Also the fractional order derivative,  $\alpha$ , is considered in the Caputo sense.

In the introduced scheme, the operational matrices of classic (non fractional) and fractional derivatives are made and employed to transform the nonlinear problem (1)-(2) into nonlinear equation. The obtained equation is discretized by fractional Bernoulli wavelets via spectral collocation and Galerkin methods. So it reduced to the nonlinear system of equations, which is solved by Newton iterative method. For analyzing the effect of fractional order on the solutions, the problem (1)-(2) has been solved for some different values of  $\alpha$ . To validate the proposed method, we have considered some illustrative examples and compared with the exact results.

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**From The First Remarkable Limit to a Nonlinear Differential Equation**

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**Abstract**

In the paper a nonlinear differential equation arising from an elementary geometry problem was discussed. This geometry problem was inspired by one of the proofs of the first remarkable limit known from the 1st semester undergraduate Calculus course. In the paper the problem was solved using an approximate geometric method which constructs broken line approximation for the curve. Compass tool of GeoGebra was extensively used for these constructions. At the end of the paper, some generalizations were discussed. A new transformation of curves, named as Interception, was introduced and its approximate construction by GeoGebra was described. It was noted in [1, Part C (Part 3 in Russian Translation), Sect. 1.370] that a differential equation of the form  $r^2 + (r')^2 = f^2(x)$  can always be transformed into I type Abel equation (see [1, Sect. 4.10], [2, Sect. 4-1]). The special case  $y^2 + (y')^2 = \frac{a^2}{\cos^4 x}$ , was discussed in [3] in relation to one kinematics problem which is dilational version of our problem (See also [1, Sect. 1.460]). Solution in quadratures for the last equation was given in [4] (See also [1, Sect. 1.460]). Some connections with problems related to laser-beam riding interception (hence the name) of high-speed missiles in defence technology was discussed in [5].

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**Global Symplectic Lanczos Method With Application To Matrix Exponential Approximation**

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**Abstract**

It is well-known that the symplectic Lanczos method is an efficient tool for computing a few eigenvalues of large and sparse Hamiltonian matrices. A variety of block Krylov subspace methods were introduced by Lopez and Simoncini to compute an approximation of  $\exp(M)V$  for a given large square Hamiltonian matrix  $M$  and a tall and skinny matrix  $V$  that preserves the geometric property of  $V$ . For the same purpose, in this paper, we have proposed a new method based on a global version of the symplectic Lanczos algorithm, called the global  $J$ -Lanczos method ( $GJ$ -Lanczos). To the best of our knowledge, this is probably the first adaptation of the symplectic Lanczos method in the global case. Numerical examples are given to illustrate the effectiveness of the proposed approach.

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**Group Action on Fuzzy Ideals of Near-Rings**

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**Abstract**

In this paper we introduce the group action on a near ring  $N$  and with it we study group action on fuzzy ideals of  $N$ ,  $G$ -invariant fuzzy ideals, finite products of fuzzy ideals and  $G$ -primeness of fuzzy ideals of  $N$ .

**Acknowledgement:** Not Applicable

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**Hadamard Products of Uniformly Starlike and Convex Functions Associated with**  
**Deniz-Özkan Differential Operator**

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**Abstract**

In this paper we introduce the subclasses  $\beta-TSP_{\lambda}^m(\alpha)$  and  $\beta-TUCV_{\lambda}^m(\alpha)$  of analytic functions defined by Deniz- Özkan Differential operator  $D_{\lambda}^m f(z)$ . We obtain modified Hadamard products of functions belonging to the subclasses  $\beta-TSP_{\lambda}^m(\alpha)$  and  $\beta-TUCV_{\lambda}^m(\alpha)$ .

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**Hardy Space of Rabotnov Function**

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**Abstract**

In this paper, we obtain conditions for the normalized Rabotnov function to belong to the Hardy space  $\mathcal{H}^\infty$ .

**References:**

1. Baricz, Á., Bessel transforms and Hardy space of generalized Bessel functions, *Mathematica*, 48(2), 127–136, 2006
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**Hilfer fractional stochastic differential equation and optimal control**

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**Abstract**

In this paper a class of Hilfer fractional stochastic differential equations is investigated. First the existence of mild solutions is studied through stochastic analysis theory, fractional calculus and semigroup theory. Next the existence of optimal pairs of the corresponding Lagrange control system is investigated. Eventually an example is presented to illustrate the results.

The control system is modelled by

$$D_{0+}^{\vartheta, \mu} x(t) = Ax(t) + f(t, x(s)) \frac{dW(t)}{dt} + \int g(t, x(t), v) \tilde{N}(dt, dv) + Bu(t), \quad t \in [0, a]$$
$$I_{0+}^{(1-\vartheta)(1-\mu)} x(0) + h(x) = x_0$$

Here  $D^{\vartheta, \mu}$  is the Hilfer fractional derivative :  $0 \leq \vartheta \leq 1, 0 < \mu < 1$ .

Hilfer [1] introduced a general operator for fractional derivative which includes Caputo and Riemann-Liouville fractional derivatives. For instance, Hilfer fractional derivative arises in the theoretical simulation of dielectric relaxation in glass forming materials. Compared to ordinary derivatives, mostly fractional derivatives are used to model many real world phenomena since the latter incorporates the hereditary and memory properties of processes and materials. In an open loop control system the aim of optimal control is to get the optimal values of control variables which optimizes the a given performance index (for more details see [2]). The existence and uniqueness of mild solution is studied by using successive approximation theory. This theory is advantageous since it involves linearization for nonlinear functional with respect to state variables. Then the existence of optimal control pairs of the dynamic system modelled by Hilfer fraction stochastic differential equation, is established by using general mild conditions of cost functional. This approach overcomes many difficulties that arise in the study of existence of solution and optimal control of stochastic control problems. Lastly the results are illustrated by an example.

**Acknowledgement:** I sincerely acknowledge the support of Mahindra University in this research.

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**Invariance of Special Class of Rational Numbers**

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**Abstract**

Consider the set of operations  $O = \left\{ T(x) = \frac{x}{2} \right\} \cup \left\{ S_k \mid S_k(x) = \frac{5x+k}{2}, k \in Z \right\}$ . Suppose that a sequence of operations  $P = B_0 B_1 \dots B_{n+m-1}$  is given, where  $B_i \in O$  for  $i = 0, 1, 2, \dots, n+m-1$ . Suppose that  $n$  is the number of  $T$  operations and  $m$  is the total number of  $S_k$  operations for all  $k$ . We assume that  $m > 0$  and  $n \geq 0$ . We will need also infinite extension of  $P$  defined by  $B_i = B_j$  if  $i \equiv j \pmod{n+m}$ . In other words, we interpret  $P$  as a sequence infinite in both directions:

$$P = \dots B_0 B_1 \dots B_{n+m-1} B_0 B_1 \dots B_{n+m-1} B_0 B_1 \dots B_{n+m-1} \dots$$

Consider the equation  $B_0 B_1 \dots B_{n+m-2} B_{n+m-1}(x) = x$ . Note that this is a linear equation and therefore its solution  $x_0$  is a rational number. We are not going to write a formula for  $x_0$ , although it is possible. We just mention the fact that  $x_0$  is completely defined by the given sequence of operations. In a similar way, let  $x_1$  be the solution of  $B_1 \dots B_{n+m-2} B_{n+m-1} B_0(x) = x$  ( $x_0 = B_0(x_1)$ ). The other numbers  $x_i$  are defined similar to these. The last number  $x_{n+m-1}$  is the solution of  $B_{n+m-1} B_0 B_1 \dots B_{n+m-2}(x) = x$  ( $x_{n+m-1} = B_{n+m-1}(x_0)$ ). Again, we can extend the numbers  $x_i$  ( $i = 0, 1, \dots, n+m-1$ ) using the equalities  $x_i = x_j$  if  $i \equiv j \pmod{n+m}$ . This means that the numbers  $x_i$  are also interpreted as a sequence infinite in both directions:

$$\dots, x_0, x_1, \dots, x_{n+m-1}, x_0, x_1, \dots, x_{n+m-1}, x_0, x_1, \dots, x_{n+m-1}, \dots$$

Let  $U_i = \frac{2^i}{2^{n+m-5m}}$ , where  $i = 0, 1, 2, \dots, n+m$ . There are infinitely many pairs of nonnegative integers  $(a, b)$ , for which  $5^a U_0 - U_b$  is an integer, or equivalently,  $5^a U_i - U_{i+b}$  is an integer ( $0 \leq i \leq i+b \leq n+m$ ). Let  $\sigma(s, r)$ , where  $s \leq r$ , be the number of all  $S_k$  operations in the fragment  $B_s B_{s+1} \dots B_{r-1}$  of the infinitely extended sequence  $P$ . In particular  $\sigma(s, s) = 0$ , because  $\sigma(s, s)$  corresponds to an empty fragment of the sequence  $P$ .

**Theorem 1.**

If the pair of nonnegative integers  $(a, b)$  satisfy  $5^a U_0 - U_b \in Z$  then for the numbers  $x_i$  defined above, the difference  $5^a x_i - 5^{\sigma(i, i+b)} x_{i+b}$  is also an integer ( $0 \leq i \leq i+b \leq n+m$ ).

**Theorem 2.**

If the pair of nonnegative integers  $(a, b)$  satisfy  $5^a U_0 + U_b \in Z$  then for the numbers  $x_i$  defined above, the sum  $5^a x_i + 5^{\sigma(i, i+b)} x_{i+b}$  is also an integer.

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**LIMIT CYCLES OF CONTINUOUS AND DISCONTINUOUS PIECEWISE DIFFERENTIAL  
SYSTEMS FORMED BY LINEAR AND CUBIC ISOCHRONOUS CENTERS**

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**ABSTRACT.** In this work, we tackle the continuous and discontinuous planar piecewise differential system separated by the straight-line  $x = 0$ ; formed by the arbitrary isochronous linear center in the half-plane  $x > 0$  and one of the following four cubic systems having an isochronous center

$$\begin{array}{ll} \dot{x} = -y + x^3 - 3xy^2, & \dot{y} = x + 3x^2y - y^3, \\ \dot{x} = -y + x^3 - xy^2, & \dot{y} = x + x^2y - y^3, \\ \dot{x} = -y + 3x^2y, & \dot{y} = x - 2x^3 + 9xy^2, \\ \dot{x} = -y - 3x^2y, & \dot{y} = x + 2x^3 - 9xy^2, \end{array}$$

in the half-plane  $x < 0$  after an affine transformation. The study of the continuous piecewise differential systems cannot provide limit cycles. In the case of discontinuous, for the first and the second cubic systems, the maximum number of limit cycles is two and one, respectively, and there are realized examples show these limit cycles; while the third and the fourth cubic systems provided at most three limit cycles, here realized examples are given with only one limit cycle.

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**Limit Cycles Of Continuous And Discontinuous Piecewise Differential Systems Formed By The  
Linear Center And The Uniform Isochronous Quadratic Or Cubic Center**

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**Abstract**

The purpose of this paper was firstly a study of the continuous planar piecewise differential systems separated by the straight-line  $y = ax + b$  formed by the linear isochronous center and the uniform isochronous quadratic or cubic center. We prove that the crossing limit cycles cannot be obtained by these piecewise differential systems. Secondly, we study also the crossing limit cycles of the discontinuous planar piecewise differential formed by the same previous differential systems.

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**Machine Learning Classification Model for Heart Failure Patients**

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**Abstract**

Heart failure is a cardiovascular disease that affects millions of people throughout the world. The goal of this study is to use multiple machine learning techniques to estimate the chance of survival of patients based on their various features, diseases, and lifestyles.

**Acknowledgement:** This research was partially supported by the Office of Scientific Research Project Coordination at the Yildiz Technical University (Grant No: FYL-2021-4416).

**Keywords:** Heart Failure, Machine Learning, Decision Tree.

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**Mathematical modeling of COVID-19 with Vaccination Strategy Driven by L'evy Jump  
Perturbation**

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**Abstract**

In this work, we study the dynamic of COVID-19 model by taking into account a vaccination strategy with L'evy jump processes. Our mathematical model incorporating four acting compartments that are the susceptibles, the infected, the vaccinated and the quarantined population. The well-posedness of our model is fulfilled by mean of proving the existence, uniqueness of the positive solution. Moreover and under some conditions, we show the extinction of infection and we give also some sufficient conditions ensuring the infection persistence case. Finally, we support the theoretical findings by the numerical simulations.



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**Mathematics Performance of Community college students in an online environment during the  
COVID-19 Pandemic**

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**Abstract**

In this session, you will dive into the effects of the COVID-19 pandemic on college students mathematics performance. We will share how students did perform in different math courses in Fall 2020 relative to different semesters.

We will share tools, practical strategies we used to differentiate instruction to meet the needs of online students during the COVID-19 pandemic.

**Acknowledgement:** New Mexico State University Student Records in the Banner System

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**Modeling the effects of density dependent emigration, weak Allee effects, and matrix hostility on patch-level population persistence**

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**Abstract**

The relationship between conspecific density and the probability of emigrating from a patch can play an essential role in determining the population-dynamic consequences of an Allee effect. In this paper, we model a population that inside a patch is diffusing and growing according to a weak Allee effect per-capita growth rate, but the emigration probability is dependent on conspecific density. The habitat patch is one-dimensional and is surrounded by a tuneable hostile matrix. We consider five different forms of density dependent emigration (DDE) that have been noted in previous empirical studies. Our models predict that at the patch-level, DDE forms that have a positive slope will counteract Allee effects, whereas, DDE forms with a negative slope will enhance them. Also, DDE can have profound effects on the dynamics of a population, including producing very complicated population dynamics with multiple steady states whose density profile can be either symmetric or asymmetric about the center of the patch. Our results are obtained mathematically through the method of sub-super solutions, time map analysis, and numerical computations using Wolfram Mathematica.

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**Neighborhoods and Partial Sums of Certain Meromorphic Functions**

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**Abstract**

In this study, using a differential operator, we define a new subclass of meromorphic functions. Some properties neighborhoods and partial sums of functions in this subclass are given.

**References:**

1. Raducanu D., Orhan, H., Erhan, D., Inclusion relationship and Fekete-Szegő like inequalities for a subclass of meromorphic functions, J. Math. Appl., 34, 87-95, 2011.
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**Neighborhoods of Certain Classes of Analytic Functions Defined By Rabotnov Function**

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**Abstract**

We introduce a new subclass of analytic functions in the open unit disk  $\mathcal{U}$  with negative coefficients defined by normalized of the Rabotnov function. The object of the present paper is to determine coefficient inequalities, inclusion relations and neighborhoods properties for Rabotnov function belonging to this subclass.

**References:**

1. Altıntaş, O., Owa, S., Neighborhoods of certain analytic functions with negative coefficients, Int. J. Math. and Math. Sci., 19, 797-800, 1996.
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**New Characterization of Schrödinger Flow with Bäcklund Transformations**

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**Abstract**

In this paper, we characterize integrable geometric Schrödinger flow with differential geometry properties of surfaces. We give some new solutions by using Bäcklund transformations. Finally, we obtain some solutions of mKdV system.

**References:**

1. NH. Abdel-All, R. A. Hussien and T. Youssef, Hasimoto Surfaces, Life Science Journal 9(3) (2012) 556.
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**New Modified Square Root Quadratic Proximal (SRQP)**

**Method for Nonlinear Complementarity Problems**

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**Abstract**

In this paper, we propose a new modified interior proximal point method for solving nonlinear complementarity problems. The method uses a profitable searching direction and instead of using the logarithmic quadratic term in the nonlinear complementarity subproblems, we used a new Square root quadratic term. We prove the global convergence of the proposed method under the assumption that  $F$  is monotone. Some preliminary computational results are given to illustrate the efficiency of the proposed method.

**Key words.** Nonlinear complementarity problems, Monotone operator, Square root quadratic term, Logarithmic quadratic term, Interior point method.

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**New type constant  $\Pi_2$ - slope curves according to type-2 Bishop frame**

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**Abstract**

In this paper, we study Smarandache  $\Pi_1\Pi_2B$  curves of biharmonic new type constant  $\Pi_2$ -slope curves according to type-2 Bishop frame in the  $SOL^3$ . Type-2 Bishop equations of Smarandache  $\Pi_1\Pi_2B$  curves are obtained in terms of base curve's type-2 Bishop invariants.

**References:**

1. L. R. Bishop: There is More Than One Way to Frame a Curve, Amer. Math. Monthly 82 (3) (1975) 246-251.
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**Numerical analysis of a diagonal hyperbolic system of non-conservative form**

**Application to dislocations densities**

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**Abstract**

In this work, we present a numerical analysis of a diagonal hyperbolic system of non-conservative form with Lipschitz initial data. Under some assumptions on the velocity of the matrix of non-conservative term of the system without supposing its strict hyperbolicity, existence and uniqueness results were proved in a work of R. Monneau and A. EL Hajj in [2]. We prove a convergence result of a finite difference scheme by taking the same assumptions like [2]. A simulation of the result is done for a system modeling dislocations densities.

**References:**

1. E. H. A. BOUDJERADA, RACHIDA AND A. OUSSAILY, *Convergence of an implicit scheme for diagonal non-conservative hyperbolic systems*, M2an, 55 (2021), pp. 573–591.
- [2] A. EL HAJJ AND R. MONNEAU, *Uniqueness results for diagonal hyperbolic systems with large and monotone data*, J. Hyperbolic Differ. Equ., 10 (2013), pp. 461–494.
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**Numerical analysis of PDE with random coefficients**

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**Abstract**

In this paper, we study the drug transport equation with random coefficients. We analyze the regularity of the solution with respect to the random variables. We prove the error estimate for the stochastic method proposed to solve our problem using the regularity result and the error estimate for the Finite difference method which is used for space discretization. Then, we provide the overall errors estimate and the convergence is achieved as a direct result. Finally some numerical results are simulated to illustrate the theoretical analysis.

**Acknowledgement:** This Project is supported by XXX-YYY.

**References:**

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**5th INTERNATIONAL ONLINE CONFERENCE ON MATHEMATICS**  
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**On (b,c)-inverses in rings**

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**Abstract**

In this talk we present left and right annihilator (b,c)-inverses and some of their properties. Furthermore, here we investigate some properties of left and right (b,c)-inverses. In addition, here we present some new results concerning (b,c)-inverses in rings.

**Acknowledgement:** This Project is supported by Ministry of Education, Science and Technological Development of Republic of Serbia (Grant No: 451-03-9/2021-14/200113).

**References:**

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**On additive formulas for the Drazin inverse of complex matrices and block representations**

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**Abstract**

In this talk we investigate a recent result concerning the Drazin inverse for the sum of two matrices. Here we show that the main result from the mentioned paper is actually a corollary of one known result. In addition, we present some new representations for the Drazin inverse of anti-triangular block matrix, which generalize some representations from current literature on the topic.

**Acknowledgement:** This Project is supported by Ministry of Education, Science and Technological Development of Republic of Serbia (Grant No: 451-03-9/2021-14/200113).

**References:**

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**On eg-Supplemented Modules**

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**Abstract**

In this work, some new properties of eg-supplemented modules are investigated. All rings have unity and all modules are unitary left modules. It is clear that every essential supplemented module is eg-supplemented. Hence eg-supplemented modules are more general than essential supplemented modules.

**Keywords:** g-Small Submodules, Essential Submodules, Supplemented Modules, g-Supplemented Modules.

**Some Results**

**Proposition 1.** Every factor module of an essential supplemented module is eg-supplemented.

**Proposition 2.** Every homomorphic image of an essential supplemented module is eg-supplemented.

**Proposition 3.** The finite sum of essential supplemented modules is eg-supplemented.

**Proposition 4.** Let  $M$  be an essential supplemented  $R$ -module. Then every finitely  $M$ -generated module is eg-supplemented.

**Proposition 5.** Hollow and local modules are eg-supplemented.

**Proposition 6.** Every g-hollow module is eg-supplemented.

**References:**

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**On Finitely e-Supplemented Modules**

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**Abstract**

In this work, some new properties of finitely e-supplemented (briefly, fe-supplemented) modules are investigated. All rings have unity and all modules are unitary left modules, in this work. Let  $M$  be an fe-supplemented  $R$ -module and  $N$  be a finitely generated submodule of  $M$ . Then  $M/N$  is fe-supplemented.

**Keywords:** Small Submodules, Essential Submodules, Supplemented Modules, f-Supplemented Modules.

**Results**

**Proposition 1.** Every f-supplemented module is fe-supplemented.

**Proposition 2.** Let  $M$  be an  $R$ -module and  $L \ll M$ . If  $M$  is f-supplemented, then  $M/L$  is fe-supplemented.

**Proposition 3.** Let  $M$  be an f-supplemented module and  $N$  be a finitely generated submodule of  $M$ . Then  $M/N$  is fe-supplemented.

**Proposition 4.** Let  $M$  be an fe-supplemented module. If every nonzero finitely generated submodule of  $M$  is essential in  $M$ , then  $M$  is f-supplemented.

**Proposition 5.** Hollow and local modules are fe-supplemented.

**References:**

1. J. Clark, C. Lomp, N. Vanaja, R. Wisbauer, Lifting Modules Supplements and Projectivity In Module Theory, Frontiers in Mathematics, Birkhauser, Basel, 2006.
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**5th INTERNATIONAL ONLINE CONFERENCE ON MATHEMATICS**  
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**On General Results on Absolute Matrix Summability Factors**

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**Abstract**

Let  $\sum a_n$  be an infinite series with its partial sums  $(s_n)$ . Let  $(\varphi_n)$  be any sequence of positive real numbers. The series  $\sum a_n$  is said to be summable  $\varphi - |A, \beta; \delta|_k$ ,  $k \geq 1$ ,  $\delta \geq 0$  and  $\beta$  is a real number, if (see [1])

$$\sum_{n=1}^{\infty} \varphi_n^{\beta(\delta k + k - 1)} |A_n(s) - A_{n-1}(s)|^k < \infty$$

where

$$A_n(s) = \sum_{v=0}^n a_{nv} s_v, \quad n = 0, 1, \dots$$

In this paper, two known theorems on absolute Riesz summability with weaker conditions are generalized for  $\varphi - |A, \beta; \delta|_k$  summability of infinite series and Fourier series.

**Keywords:** Fourier series, infinite series, absolute matrix summability

**References:**

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**5th INTERNATIONAL ONLINE CONFERENCE ON MATHEMATICS**  
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**On Rough Statistical Convergence**

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**Abstract:** In this study, we analyze the rough convergence and some generalizations of it.

The notion of statistical rough convergence and rough ideal convergence are examined in particular, and the results obtained are compared. Furthermore, we also investigate a new generalization of the notion of rough ideal convergence.

**Keywords:** Rough Convergence, Rough Statistical Convergence, Rough Ideal Convergence, Rough Ideal Statistical Convergence.

**References:**

- [1] H. Fast, “Sur la convergence statistique”, Colloq. Math. 2(1951), 241-244.
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**On spectral Petrov-Galerkin method for solving fractional initial value problems in weighted Sobolev space**

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**Abstract**

In this paper, we investigate a spectral Petrov-Galerkin method for fractional initial value problems. Singularities of the solution at the origin inherited from the weakly singular kernel of the fractional derivative are considered, and the regularity is constructed for the solution in weighted Sobolev space. We present an optimal error estimate of the spectral Petrov-Galerkin method, and prove that the convergence order of the method in the weighted  $L^2$ -norm is  $3\alpha + 1$  for smooth source term, where  $\alpha$  is the order of the fractional derivative. An iteration algorithm with a quasi-linear complexity is considered to solve the produced linear system. Numerical experiments verify the theoretical findings and show the efficiency of the proposed algorithm, and exhibit that the presented numerical method works well for some time-fractional diffusion equations after suitable temporal semi-discrete.

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**On s-Supplemented Modules**

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**Abstract**

In this work, some new properties of socle-supplemented (briefly, s-supplemented) modules are studied. Every ring has unity and every module is unitary left module, in this work. It is proved that every factor module and every homomorphic image of an s-supplemented module are s-supplemented.

**Keywords:** Small Submodules, Radical, Socle, Supplemented Modules.

**Some Results**

**Proposition 1.** Every s-supplemented module is essential supplemented.

**Proposition 2.** Every factor module of an s-supplemented module is essential supplemented.

**Proposition 3.** Every homomorphic image of an s-supplemented module is essential supplemented.

**Proposition 4.** The finite sum of s-supplemented modules is s-supplemented.

**Proposition 5.** The finite direct sum of s-supplemented modules is s-supplemented.

**Proposition 6.** Hollow and local modules are s-supplemented.

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**5th INTERNATIONAL ONLINE CONFERENCE ON MATHEMATICS**  
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**On statistical convergence modulated by a function**

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**Abstract**

Recently, the study of statistical convergences more sophisticated than the classical one, have aroused the interest of several researchers. Such is the case in which the density in the set of natural numbers is modulated by means of a function  $f$ , increasing on  $[0, \infty)$  and continuous. This new way to compute the density of natural numbers leads to new convergence methods:  $f$ -statistical convergence. In this lecture we will discover the properties of the modulus functions  $f$  for classical results by Connors and Khan-Orhan which are true for the classical statistical convergence, remains true for the  $f$ -statistical convergence.

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**On Strongly  $\oplus$ -g-Rad-Supplemented Modules**

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**Abstract**

In this work, all rings have unity and all modules are unitary left modules. It is investigated some new properties of strongly  $\oplus$ -g-Rad-supplemented modules, in this work. Let  $M$  be a strongly  $\oplus$ -g-Rad-supplemented  $R$ -module. If  $M$  is supplemented, then  $M$  is strongly  $\oplus$ -supplemented.

**Keywords:** Essential Submodules, g-Small Submodules, Supplemented Modules, g-Supplemented Modules.

**Results**

**Proposition 1.** Let  $M$  be a strongly  $\oplus$ -g-Rad-supplemented module. Then  $M$  is g-semilocal.

**Proposition 2.** Every strongly  $\oplus$ -g-supplemented module is strongly  $\oplus$ -g-Rad-supplemented.

**Proposition 3.** Let  $M$  be a g-supplemented module. If every g-radical supplement submodule in  $M$  is a direct summand of  $M$ , then  $M$  is strongly  $\oplus$ -g-Rad-supplemented.

**Proposition 4.** Every strongly  $\oplus$ -g-Rad-supplemented module is  $\oplus$ -g-Rad-supplemented.

**Proposition 5.** Let  $M$  be a g-radical supplemented module and  $K \leq M$ . If every g-radical supplement submodule in  $M/K$  is a direct summand of  $M/K$ , then  $M/K$  is strongly  $\oplus$ -g-Rad-supplemented.

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**5th INTERNATIONAL ONLINE CONFERENCE ON MATHEMATICS**  
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**On Summability of Infinite Series and Fourier Series**

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**Abstract**

Let  $A = (a_{nv})$  be a normal matrix, i.e., a lower triangular matrix of non-zero diagonal entries. Let  $\sum a_n$  be an infinite series with its partial sums  $(s_n)$ . The series  $\sum a_n$  is said to be summable  $|A, p_n, \beta; \gamma|_k$ ,  $k \geq 1$ ,  $\gamma \geq 0$  and  $\beta$  is a real number, if (see [1])

$$\sum_{n=1}^{\infty} \left( \frac{p_n}{p_n} \right)^{\beta(\gamma k + k - 1)} |A_n(s) - A_{n-1}(s)|^k < \infty$$

where  $(p_n)$  is a sequence of positive numbers such that

$$P_n = \sum_{v=0}^n p_v \rightarrow \infty \text{ as } n \rightarrow \infty \quad (P_{-m} = p_{-m} = 0, m \geq 1)$$

and

$$A_n(s) = \sum_{v=0}^n a_{nv} s_v, \quad n = 0, 1, \dots$$

A sequence  $(c_n)$  is said to be  $\delta$ -quasi-monotone if  $c_n \rightarrow 0$ ,  $c_n > 0$  ultimately and  $\Delta c_n \geq -\delta_n$ , where  $\delta = (\delta_n)$  is a sequence of positive numbers [2]. In this paper, two known theorems on absolute Riesz summability factors of infinite and Fourier series are generalized to  $|A, p_n, \beta; \gamma|_k$  summability method by using  $\delta$ -quasi-monotone sequences.

**Keywords:**  $\delta$ -quasi-monotone sequences, infinite series, Fourier series

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**On the Maximal Output Admissible Set for a Class of Bilinear Discretetime Systems**

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**Abstract**

Given a discrete-time controlled bilinear systems with initial state  $x_0$  and output function  $y_i$ , we investigate the maximal output set  $\Theta(\Omega) = \{x_0 \in \mathbb{R}^n : y_i \in \Omega, \forall i \geq 0\}$  where  $\Omega$  is a given constraint set and is a subset of  $\mathbb{R}^p$ . Using some stability hypothesis, we show that  $\Theta(\Omega)$  can be determined via a finite number of inequations. Also, we give an algorithmic process to generate the set  $\Theta(\Omega)$ . To illustrate our theoretical approach, we present some examples and numerical simulations. Moreover, to demonstrate the effectiveness of our approach in real-life problems, we provide an application to the SI epidemic model and the SIR model.

**Acknowledgement:** Research reported in this paper was supported by the Moroccan Systems Theory Network..

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**On the motion of a dumbbell micro-spheres subjected to a time-periodic external field**

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**Abstract**

The motions of a dumbbell micro-spheres in a viscous incompressible fluid at low Reynolds number are investigated analytically. The dumbbell micro-spheres consist of two rigid spheres connected by a spring; one of them is non-conducting sphere and the other is conducting sphere moved under the action of an external oscillator field. The fluid flow past the micro-spheres is described by the Stokes equation and is solved analytically using the two-timing method. A systematic description of the average velocity of the system is provided. The results demonstrated that the system moves in a circular path with a speed that decreases inversely with the length of the spring.

**Key words:** fluid dynamics, low Reynolds number, oscillation motion, Stokes equation, two-timing method.

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**On The Ruled Surface According to Dual Bézier Curves**

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**Abstract**

The set of dual numbers is defined by  $D = \{a + \varepsilon a^*: a, a^* \in R, \varepsilon \neq 0; \varepsilon^2 = 0\}$ . Let  $D^3$  be the set of dual vectors stated as  $D^3 = \{u = (u_1, u_2, u_3): u_i \in D, i = 1, 2, 3\}$ . In this paper the ruled surface corresponding a dual Bézier curve is studied. The invariants of this surface are introduced.

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**5th INTERNATIONAL ONLINE CONFERENCE ON MATHEMATICS**  
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**On the semiprime submodules of Noetherian modules**

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**Abstract**

Let  $R$  be a commutative ring and  $M$  be an  $R$ -module. A proper submodule  $S$  of  $M$  is called semiprime if for every submodule  $K$  of  $M$  and every ideal  $I$  of  $R$ ,  $I^2 K \subseteq S$  implies that  $IK \subseteq S$ . In this talk we consider semiprime submodules over Noetherian modules and give some results

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**On the Spectral Properties of a Boundary Value Problem with Spectral Parameters**

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**Abstract**

In the present paper, we consider boundary value problem with a spectral parameter in the equation, of the form

$$-u'' + q(x)u = \lambda^2 u, \quad 0 < x < 1, \quad (1)$$

$$(\alpha_0 + \alpha_1 \lambda) u(0) + u'(0) = 0, \quad (2)$$

$$(\beta_0 + \beta_1 \lambda) u(1) + u'(1) = 0, \quad (3)$$

where  $\lambda$  is a spectral parameter,  $q(x)$  is a nonnegative continuous function on the interval  $[0,1]$  and  $\alpha_i$  and  $\beta_i$  are real constants ( $i=0,1$ .)

Some versions of the problem for equation (1) were studied and different statements were found in [1-3].

In this study, the eigenvalues of the boundary value problem (1)-(3) containing spectral parameters at the boundary and the eigenfunctions corresponding to these eigenvalues are examined. Some properties of these eigenvalues and the oscillation theorem about the zeros of the eigenfunctions is proved.

**Keywords: : Eigenvalue, Eigenfunction, Oscillation Theorem.**

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**5th INTERNATIONAL ONLINE CONFERENCE ON MATHEMATICS**  
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**On The Spherical Projection of Dual Bézier Curves**

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**Abstract**

The set of dual numbers is defined by  $D = \{a + \varepsilon a^* : a, a^* \in R, \varepsilon \neq 0; \varepsilon^2 = 0\}$ . Let  $D^3$  be the set of dual vectors stated as  $D^3 = \{u = (u_1, u_2, u_3) : u_i \in D, i = 1, 2, 3\}$ . In this paper the projection curve of any curve given in the dual vector space  $D^3$  to the dual unit sphere is studied.

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**On Wijsman  $I_\lambda$ -Statistical Convergence for Sequences of Sets in Intuitionistic Fuzzy Normed Spaces**

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**Abstract**

In this study, we investigate the notions of Wijsman  $I - [V, \lambda]$  –summability and Wijsman  $I_\lambda$  –statistical convergence for sequences of sets with regards to the intuitionistic fuzzy norm (briefly, IFN)  $(\mu, \nu)$ , examine their relationship, and make some observations about these classes. We mainly study the relation between these two new methods and the relation between Wijsman  $I - \lambda$  –statistical convergence and Wijsman  $I$  – statistical convergence for sequences of sets in the corresponding intuitionistic fuzzy normed space.

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**Online Professional Development in STEM Education during the COVID-19 Pandemic: STEM Teacher Institutes**

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**Abstract**

We designed an online professional development program to help teachers develop themselves professionally in STEM education during the COVID-19 pandemic. The program set principles and standards for online professional development programs in STEM education during outbreaks. In line with those principles and standards, we presented the program to teachers. This paper aimed to determine the impact of the program on the professional development of teachers. Data were collected using a semi-structured interview form. The data were analyzed using content analysis, and themes, categories, and codes were developed. Participants had positive views of the program and stated that they would like to participate in professional development programs not only during outbreaks but also routinely. They had faced numerous challenges during the program, and therefore, remarked that teachers should be trained on technological literacy before attending online professional development programs. We think that the online professional development program we designed sets the principles and standards for future programs.

**Keywords:** Teacher, pandemic, STEM education, online professional development

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**Partial asymptotic null-controllability for semilinear evolution equations in Hilbert spaces**

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**Abstract**

This work deals with the problem of partial asymptotic null controllability for constrained systems governed by semi-linear partial differential equations, where linear part generates a strongly continuous compact semigroup on Hilbert spaces. This consists of driven only a desired part of the system's state to the origin taking into account mixed state-input constraints. We give sufficient conditions for the existence of appropriate control laws to ensure partial asymptotic null-controllability. The explicit expression of such controls is given as strongly-weakly continuous selections of a set-valued map which is defined through a certain practical tangential condition. Application to semilinear parabolic partial differential equations is treated in order to illustrate the results established.

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**Partial asymptotic null-controllability with mixed state-input constraints**

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**Abstract**

This paper is devoted to the problem of partial asymptotic null-controllability of control systems governed by ordinary differential equations, subjected to possibly mixed state-input constraints. Using Lyapunov functions within the framework of viability theory, feedback controls are designed in such a way a part of system's state can be driven to the origin asymptotically, taking into account the mixed constraints. By using Michael selection theorem, the existence of such controls is proved, in the case of convex constraints, and their expressions are given as continuous selections of an appropriate constructed multifunction. Finally, two examples are processed numerically in order to illustrate the theoretical results

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**Poisson algebras and Poisson prime ideals**

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**Abstract**

The concept of Poisson algebra is one of the most important concepts in mathematics that make a link between commutative and noncommutative algebra. The Poisson algebra can be defined as a Lie algebra that satisfies the Leibniz rule. In this talk, I will give the definition of the Poisson algebra, talk about some properties of Poisson algebras, Poisson prime ideals, Poisson spectra, simple Poisson algebras, Skew polynomial Poisson algebras and Generalized Weyl Poisson algebras.

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**Prime Ideals of Gamma Nearness Near-Ring**

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**Abstract**

The concept of  $\Gamma$ -rings, a generalization of a ring was introduced by Nobusawa in 1964 and generalized by Barnes. Pilz defined near-rings (also near ring or nearring) that is an algebraic structure similar to a ring but satisfying some axioms. A generalization of both the concepts near-ring and the ring, namely  $\Gamma$ -near-ring was introduced by Satyanarayana in 1984 and later studied by many authors. In 2002, Peters introduced near set theory, which is a generalization of rough set theory. In this theory, Peters defined an indiscernibility relation by using the features of the objects to determine the nearness of the perceptual objects. Perceptual objects (nonabstract points) can be used on weak nearness approximation space to define nearness algebraic structures. This is more useful than working with abstract points for many areas such as engineering applications, image analysis and so on. In 2012, İnan and Öztürk investigated the concept of nearness groups and other algebraic approaches of near sets. In 2021, Uçkun and Genç defined near-rings on nearness approximation spaces. Also, Tekin introduced  $\Gamma$ -nearness near-ring on weak approximation spaces in 2021. The aim of this paper is to define the notion of prime ideals of  $\Gamma$ -nearness near-rings. Besides, it is explained that some of the concepts and definitions .

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**Problems and results in the matching problem for max-plus dynamical systems**

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**Abstract**

Linear dynamical systems over the max-plus algebra  $R_{\max}$ , or max-plus dynamical systems, provide a suitable formalism to study control properties of a class of discrete event systems (DES) that can be viewed as timed event graphs, that is as Petri nets where all the places have only one upstream and one downstream transition. Systems of this kind are important in many applications, especially in industrial engineering, and a number of analysis and control techniques have been developed in the last years. In this talk, we consider the problem of forcing a given plant, modeled as a linear system over  $R_{\max}$ , to generate an output that equals that of a given model. We tackle this problem, called the model matching problem, by means of a structural geometric approach and we provide and discuss solvability conditions. Then, we describe an extension of the same problem to the case in which the dynamical structure of the plant is not fixed, but it changes at the occurrence of specific events, giving rise to a so-called switching system. Differences with the non-switching case are highlighted and a number of results about solvability of the problem are discussed.

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**Pseudo Concircular Ricci Symmetric Spacetimes Admitting Special Conditions**

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**Abstract**

This paper deals with pseudo concircular Ricci Symmetric spacetimes with some special conditions. In the first section, we give the definition of pseudo concircular Ricci symmetric manifold  $(PCRS)_n$ . In the second section, some properties of the Z-symmetric tensor are mentioned. In the third section, we consider this tensor on  $(PCRS)_n$  manifold and we give some theorems. In the last section, considering some special conditions, we discuss the properties of these spacetimes.

**Acknowledgement:**

**References:**

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**Quasi Focal Curves of Timelike Curves in Minkowski Space**

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**Abstract**

In this study, we firstly characterize focal curves by considering quasi frame in the ordinary space. Then, we obtain the relation of each quasi curvatures of curve in terms of focal curvatures. Finally, we give some new conditions with constant quasi curvatures in the ordinary space.

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**Reducibility of some generalized principal series of the metaplectic group**

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**Abstract**

We determine reducibility of the representation of the metaplectic group induced from the tensor product of an essentially square integrable representation attached to the Zelevinsky segment and a genuine cuspidal representation of the metaplectic group.

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**Robust parallel solver for computational continuum mechanic problems**

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**Abstract**

Multigrid methods are used in software for solving problems of the computational continuum mechanics [1]. This paper represents Robust Multigrid Technique (RMT) for segregated/coupled solving the systems of nonlinear partial differential equations (multiphysics) in black-box software [2]. The solver properties we discuss are:

- robustness (the least number of problem-dependent components);
- efficiency (close-to-optimal algorithmic complexity);
- parallelism (faster than the best sequential algorithm).

Single grid RMT uses essential multigrid principle  $h$ -independent convergence. The most powerful coarse grid correction strategy makes it possible to minimize the number of problem-dependent components [3]. Special attention is paid to parallel treatment of RMT using OpenMP technology [4]. Large-scale granularity of the parallel RMT (geometric parallelism) coupled with the multicolored Gauss-Seidel iterations (algebraic parallelism) lead to almost full parallelism of the multigrid iterations. The geometric approach based on a decomposition of the given problem into a number of subproblems without an overlap is used to overcome the problems of large communication overhead and idling processors on the very coarse grids.

**Acknowledgement:** The work was supported by Russian Foundation for Basic Research, Grant 21-51-46007 («Development and application of highly efficient parallel algorithms for supercomputer modeling of complex reacting flows»), and Scientific and Technological Research Council of Turkey (TÜBİTAK), Grant No: ARDEB-220N170.

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# **Second Order parallel tensor on generalized f.pk-space form and hypersurfaces of generalized f.pk-space form**

**Punam Gupta** and Sanjay Kumar Singh

**Abstract.** The purpose of the present paper to study a second order symmetric parallel tensor in generalized f.pk-space form. Second order symmetric parallel tensor in f.pk-space form is combination of the associated metric tensor and 1-forms of structure vector fields. We prove that there does not exist second order skew-symmetric parallel tensor in f.pk-space form. We also deduce that there is no parallel hypersurface in a generalized f.pk-space form but there is semi-parallel hypersurfaces in a generalized f.pk-space form.

**MSC2020:** 53D10, 53C25, 53C21.

**Keywords:** f.pk-space form, parallel tensor, parallel hypersurface, semi-parallel hypersurface.

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**Semi-recursive kernel conditional density estimators  
under random censorship and dependent data**

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**Abstract**

In this work, we extend to the case of the strong mixing data the results of Khardani and Semmar. A kernel-type recursive estimator of the conditional density function is introduced. We study the properties of these estimators and compare them with Rosemblatt's nonrecursive estimator. Then, a strong consistency rate as well as the asymptotic distribution of the estimator are established under an  $\alpha$ -mixing condition. A simulation study is considered to show the performance of the proposed estimator.

**Acknowledgement:** This Project is supported by XXX-YYY.

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**Several New Bounds of Gauss-Jacobi Type Quadrature Formula Pertaining to s-Convex Functions**

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**Abstract**

In this paper, authors found a new interesting integral identity regarding Gauss-Jacobi type quadrature formula using generalized fractional integral operators. By using this identity as an auxiliary result, some new bounds with respect to Gauss-Jacobi type quadrature formula pertaining to s-convex functions are established. It is pointed out that several special cases are deduced from the main results for suitable choices of function inside the generalized fractional integral operators. Some basic fractional integral operators of important interest that we investigated in details are Riemann-Liouville fractional integral operator, Riemann-Liouville fractional integral operator and conformable fractional integral operator. The Gauss-Jacobi type quadrature formula has remained an area of great interest due to its wide applications in the field of mathematical analysis. We believe that this new results are crucial and of great interest for interested readers working in the fields of inequalities, fractional calculus, quantum calculus, numerical analysis and applied mathematics. These ideas and techniques of this paper may stimulate further research in these directions for different class of functions.

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**Several New Bounds of Hermite-Hadamard Type Integral Inequalities Pertaining to s-Convex Functions And Their Applications**

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**Abstract**

In this paper, authors found a new result regarding Hermite-Hadamard type integral inequalities using generalized fractional integral operators. Furthermore, a new interesting integral identity about Hermite-Hadamard type integral is derived. By using this identity as an auxiliary result, some new bounds with respect to Hermite-Hadamard type integral inequalities pertaining to s-convex functions are established. It is pointed out that several special cases are deduced from the main results for suitable choices of function inside the generalized fractional integral operators. In order to show the efficient of our main results, some applications to special means for different positive real numbers and error bound estimates for trapezoidal quadrature formula are obtain as well.

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**Skorokhod reflection problem for regulated processes.**

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**Abstract**

In this talk, we introduce a new Skorokhod problem when the input has only right and left limits. We show the existence and uniqueness of this problem. Then, we apply our result to show the existence and uniqueness of solutions of reflected Stochastic differential equations driven by semimartingales with regulated trajectories.

**Keywords:** Reflection, Skorokhod problem, Reflected stochastic differential equations.

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**Smarandache TNB Curves of Helices in Sol Space**

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**Abstract**

In this paper, we define new Smarandache TNB curves of helices in the  $Sol^3$ . We obtain parametric and vector equations of Smarandache TNB curves.

**References:**

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**Solution for Second-Order Differential Equation Using Least Square Method**

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**Abstract**

This paper study the numerical method for solving differential equation. The least square method (LSM) alongside with the  $L_2$  norm are used to obtain explicit solution and the minimum approximation error respectively.

Keywords: Differential Equation, Least Square Method and  $L_2$  norm.

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**Solving Lientof model with algebraic methods and its implementation in R**

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**Abstract**

In economics an important issue is the balance between the production of the main sectors of economy and the external demand of the production. Refereeing to the Leontief model, the structure of each industry's production activity is represented by appropriate structural coefficients that describe relationships between the inputs that the industry absorbs and the output that it produces. We study the economic development of Albania for ten-year period. We solve the problem in different algebraic ways using matrices theory. Then, we implement the solutions in R programming language and analyze the final results.

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**Some Combinatorial Properties of Spanning Simplicial Comple**

**Nazeran Idrees**

**Abstract**

This work will deal with the generalization of f-vector, shellability, Stanley-Reisner ideal, facet ideal, Alexander dual and number of facets of different classes of simplicial complexes. In this thesis, we characterize some combinatorial and algebraic properties of spanning simplicial complex  $\Delta s(Bm,n)$  of bi-cyclic graph  $Bm,n$ , where two cycles are joined by an edge for both the cases when length of both cycles is same and when length of both cycles is different. We will discuss the edge set for this graph. We will compute all spanning trees of the bi-cyclic graph  $Bm,n$  and will give the general formula for f-vector of the spanning simplicial complex  $\Delta s(Bm,n)$  particularly and consequently a formula for the Hilbert-series of the Stanley-Reisner ring  $K[\Delta s(Bm,n)]$  Where  $K$  is a field. Finally, we compute the formula for h-vector. We will also prove that the corresponding simplicial complex is shifted and as a conclusion is shellable. We analysed spanning simplicial complex of another type of bi-cyclic graph that is the bi-cyclic graph with a common vertex  $B'm,n$  where length of both cycles is taken to be different. We find f-vector, h vector, Hilbert-series and Stanley-Reisner ring of the spanning simplicial complex  $\Delta s(B'm,n)$  of bi cyclic graph with a common vertex. As a closeup we will end with the result that this spanning simplicial complex is also shellable.

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**Some Convexity Properties for a New  $p$  – valent Integral Operator**

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**Abstract**

In this paper, we define a new general  $p$  – valent integral operator and obtain the properties of convexity of this integral operator of  $p$  – valent function on some subclasses of analytic functions.

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**Some Families of Meromorphic Functions Involving a Differential Operator**

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**Abstract**

Let  $\Sigma$  denote the class of functions of the form  $f(z) = \frac{1}{z} + \sum_{k=0}^{\infty} a_k z^k$  which are analytic in the punctured disc  $\mathbb{D} = \{z : 0 < |z| < 1\}$ . We introduce and study some new families of meromorphic functions defined by a differential operator. A number of useful characteristics of functions in these families are obtained.

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**Some measures of dependence in the case of Sub-Gaussian  
symmetric alpha-stable random vectors**

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**Abstract**

Sub-Gaussian alpha-stable distributions are a particular sub-class of multivariate alpha-stable distributions, which have been used in fields such as finance and signal processing. For these particular distributions, we specify three measures of dependence proposed with the aim to quantify the dependence between the components of a symmetric alpha-stable random vector: the codifference, the generalized association parameter and the signed symmetric covariation coefficient and state a relation between these three measures. We also establish a relation which allows us to estimate the generalized association parameter without a previous estimation of the spectral measure.

**Keywords:** Codifference, Covariation, Generalized association parameter, Sub-Gaussian alpha-stable random vector.

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**Some Properties of Cofinitely eg-Supplemented Modules**

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**Abstract**

In this work, some new properties of cofinitely eg-supplemented modules are studied. Every ring has unity and every module is unitary left module, in this work. It is clear that every cofinitely essential supplemented module is cofinitely eg-supplemented. Because of this, cofinitely eg-supplemented modules are more general than cofinitely essential supplemented modules.

**Keywords:** Cofinite Submodules, Essential Submodules, Cofinitely Supplemented Modules, g-Supplemented Modules.

**Some Results**

**Proposition 1.** Every cofinitely supplemented module is cofinitely eg-supplemented.

**Proposition 2.** Let  $M$  be a cofinitely supplemented module. Then every  $M$ -generated module is cofinitely eg-supplemented.

**Proposition 3.** Every essential supplemented module is cofinitely eg-supplemented.

**Proposition 4.** Every supplemented module is cofinitely eg-supplemented.

**Proposition 5.** Hollow and local modules are cofinitely eg-supplemented.

**Proposition 6.** Every g-hollow module is cofinitely eg-supplemented.

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**Some properties of Finite Generalized Groups**

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**Abstract**

Generalized groups was introduced by M.R. Molaei in 1999, as an extension of the groups. It has a background in Unified Gauge Theory. We will review of Generalized groups. In this article, we consider the Generalized groups in finite state, they have interesting properties, some of these properties research.

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**Some Results on Rough Weighted Ideal Statistical Convergence of Sequences**

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**Abstract**

In this study, we consider and examine rough weighted  $I$ -statistical limit set and weighted  $I$ -statistical cluster points set which are natural generalizations of rough  $I$ -limit set and  $I$ -cluster points set respectively. To highlight the variation from essential results we place into some new examples. So our aim is to analyze the different behaviors of the new convergences and describe both the sets with topological approach like closedness, boundedness, compactness etc.

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**Some Special Types of Legendre Curves**

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**Abstract**

In this study, triharmonic Legendre curves of constant torsion in three dimensional Sasakian space forms are discussed. First of all, the existence of such curves in three dimensional Sasakian space forms was investigated. After the main theorem regarding the existence is given, the necessary and sufficient conditions are obtained. Finally, these conditions are examined under some special cases.

**Key words:** Legendre curves, Polyharmonic curves, Sasakian space forms

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**Stability for generalized proportional Caputo fractional differential equations**

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**Abstract**

Fractional calculus has recently acquired plentiful circulation and great significance because of its wide applications in fields of science and engineering. In recent years, there are honorable efforts have been exhausted for obtaining new classes of fractional operators by introducing more general or new kernels. Among these new definitions is the generalized proportional derivative including exponential function in the kernel. It is well-behaved and it generalizes the standing Riemann-Liouville and Caputo fractional integrals and derivatives in the literature. In this paper we study nonlinear Caputo fractional differential equations with generalized proportional derivative of the type:

$${}^C D_t^{\alpha, \rho} x(t) = f(t, x(t)), \quad \text{for } t > t_0$$

where  $\alpha \in (0, 1)$ ,  $\rho \in (0, 1]$ ,  $x \in \mathbb{R}^n$ ,

$${}^C D_t^{\alpha, \rho} x(t) = \frac{1}{\rho^{1-\alpha} \Gamma(1-\alpha)} \int_{t_0}^t e^{\frac{\rho-1}{\rho}(t-s)} (t-s)^{-\alpha} [(1-\rho)x(s) + \rho x'(s)] ds$$

One of the main qualitative properties of the solutions of any kind of differential equations is stability. One of the methods of the investigation stability is the second method of Lyapunov. The most applicable functions are quadratic ones. In connection with this we prove some properties of the generalized proportional fractional derivatives of quadratic Lyapunov functions. Also, some sufficient conditions for stability are obtained.

**Acknowledgement:** This research is supported by the Bulgarian National Science Fund under Project KP-06- N32/7.

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**Statistical Convergence and Some Generalization**

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**Abstract**

In this article, we consider the notion of statistical convergence which is based on the notion of the natural density of positive integers. Our aim in this paper is to study generalizations of statistical convergence. The relations between these generalizations and statistical convergence are examined. Moreover, we also investigate statistical analogues of some well-known basic results in classical analysis.

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**Statistical inference for non-ergodic weighted fractional Vasicek models**

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**Abstract**

We study a problem of drift parameter estimation for a non-ergodic weighted fractional Vasicek model defined as  $dX_t = \theta(\mu + X_t)dt + dB_t^{a,b}$ ,  $t \geq 0$  with unknown parameters  $\theta > 0$ ,  $\mu \in \mathbb{R}$  and  $\alpha := \theta\mu$ , whereas  $B^{a,b} := \{B_t^{a,b}, t \geq 0\}$  is a weighted fractional Brownian motion of parameters  $a > -1$ ,  $|b| < 1$ ,  $|b| < a + 1$ . We provide least square-type estimators  $(\hat{\theta}_T, \tilde{\mu}_T)$  and  $(\hat{\theta}_T, \tilde{\alpha}_T)$ , respectively, for  $(\theta, \mu)$  and  $(\theta, \alpha)$  based a continuous-time observation of  $\{X_t, t \in [0, T]\}$  as  $T \rightarrow \infty$ . The strong consistency and the joint asymptotic distribution of  $(\hat{\theta}_T, \tilde{\mu}_T)$  and  $(\hat{\theta}_T, \tilde{\alpha}_T)$  are studied. Moreover, we obtain that the limit distribution of  $\hat{\theta}_T$  is a Cauchy-type distribution, and  $\tilde{\mu}_T$  and  $\tilde{\alpha}_T$  are asymptotically normal.

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**Statistical Physics Approach to Small Scale Artificial Neural Networks**

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**Abstract**

We discuss the possible statistical approach to small-scale artificial neural networks (ANNs). We investigate different alternative feedback algorithms (gradient methods, target attractor feedback) to train ANN with data collected from the 'Ab initio' principle, similar to the model proposed by Wang, Jiang, and Zhou in 2020. The well-trained ANNs could be applied for efficient modeling of different physical systems: spin structures, phase transitions, and other related statistical systems.

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**Stochastic Ill-posed Problem with  $\alpha$ -mixing errors**

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**Abstract**

In this work we consider the linear ill posed problem described by the operator equation  $Ax = u$ , in Hilbert space in which the second member is measured with  $\alpha$ -mixing errors.

To solve this problem we propose a stochastic procedure of Robbins-Monro type which converges almost completely to the exact solution. To check the validity of our results, we consider some numerical examples.

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**Ternary Hom-Jordan algebras induced by Hom-Jordan algebras**

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**Abstract**

The purpose of the talk is to study the relationships between a Hom-Jordan algebras and its induced ternary Hom-Jordan algebras. We introduce the notion of  $\alpha$  k -derivation,  $\alpha$  k - quasiderivation and generalized  $\alpha$  k -derivation of ternary Hom-Jordan algebras, and we give some construction of ternary Hom-Jordan algebras.

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**Survey of stability for various types of fractional differential equations**

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**Abstract**

Some types of fractional differential equations with fractional derivatives such as Caputo fractional derivative, Riemann-Liouville fractional derivative, generalized proportional fractional derivative are presented. Some stability properties of the solutions will be discussed and compared. The similarity as well as the differences will be pointed out. Some examples will illustrate the theoretical study.

**Acknowledgement:** This research is supported by the Bulgarian National Science Fund under Project KP-06- N32/7.

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**Symmetric Fuzzy Stochastic Differential Equations: Generalization of the Lipschitz Condition**

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**Abstract**

In the communication we will consider stochastic differential equations with values in the space of fuzzy sets. This kind of stochastic equation may be suitable for modelling phenomena subjected to random factors and whose values are not precisely described numerically. The properties of fuzzy sets mean that, unlike the classic stochastic form of differential equations, it now makes sense to consider such equations in a certain symmetric form. The talk will present the results concerning a theorem on existence and uniqueness of solution, assuming that the coefficients of the equation satisfy a different condition than the classic Lipschitz condition. Keywords: Fuzzy stochastic differential equation, modelling in fuzzy and random environment.

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**Tangent Surfaces of Adjoint Curves**

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**Abstract**

In this study, we define new surfaces of adjoint curves as tangent surface. Also, we construct new characterizations for tangent surfaces of adjoint curves.

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**The 2-D Hyper-complex Gabor Quadratic-Phase Fourier Transform and Uncertainty Principles**

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**Abstract**

In this paper, we present a novel integral transform known as the 2-D hyper-complex(quaternion) Gabor quadratic-phase Fourier transform (Q-GQPFT), which is embodiment of several well known signal processing tools. We first define the 2-D hyper-complex(quaternion) quadratic-phase Fourier transform (Q-QPFT) and then we propose the definition of novel Q-GQPFT, which is a modified version of the classical windowed quadratic-phase Fourier transform to quaternion-valued signals and we study various properties of the proposed Q-GQPFT, including Moyal's formula, reconstruction formula, isometry and reproducing kernel formula. We also establish the Heisenberg and logarithmic uncertainty inequalities for the Q-GQPFT.



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**The algebraic method of solution of a inhomogeneous linear system of differential equations with constant coefficients and its implementation in R**

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**Abstract**

In differential equation the linear systems are very important to model different situations of life day problems. It is more convenient to direct the problem of solving a linear system of first order of differential equations to linear algebra. We consider the system  $\frac{dx}{dt} = Ax + h(t)$ . First, we find the fundamental matrix, which is the solution of the corresponding homogeneous system. The matrix  $A$  must be diagonalizable, condition that allow us to transform the given system in two parts that simplify the solution of the problem. Then we consider the exponential method to arrive at the solution. We use R programming language to implement the solution of the linear system and to interpret the independence of the solution on its variable  $t$ .

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**The Analytic Solutions for 2D First Kind Fredholm Integral Equations with Symmetric Kernels  
Using Eigenfunction Expansion**

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**Abstract**

As we know, the first kind Fredholm integral equations (FIEs) are ill-posedness and there is no definite method determining their analytic solutions. In this paper, the method of eigenfunction expansion is considered for obtaining analytic solutions of two dimensional first kind FIEs. Finally, some examples are given to check the effectiveness and simplicity of this method.

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**The Nehari Manifold Method For Perturbed Dynamical Systems**

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**Abstract**

We study the existence and multiplicity of solutions to a class of perturbed dynamical systems. The main tool is the use of Nehari manifold method, which allow to give some analysis techniques to guarantee that the system of our study have some results on the existence and multiplicity of solutions.

**The Source of Semiprimeness on Semigroup Types**

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**Abstract**

In this study, some properties of the source set of semiprimeness defined as  $S_S = \{ a \in S : aSa = 0 \}$  for the types semigroups  $S$  will be examined. The aim of this study is to get effective solutions to problems with generalizations and connections to be obtained in semigroup theory which is one of the important subjects of mathematics and especially algebra. In addition, investigating this subject in semigroup types will both add new definitions and theorems to the literature, and help establish the relationships between ring theory and semigroup theory.

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**The Spectral Expansion Formula for a Discontinuous Equation of Second Order**

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**Abstract**

In this study, an inverse scattering problem is considered for a discontinuous Sturm-Liouville equation on the half-line  $[0, \infty)$  with a linear spectral parameter in the boundary condition. Special solutions and scattering data are defined. The resolvent operator is constructed and the spectral expansion formulas in terms of scattering data are obtained. Some versions of the problem are studied in [1,2] and necessary and sufficient conditions characterizing the spectral data of the similar boundary value problems are given in [3,4].

**Keywords:** Singular differential operator, spectral parameter, expansion formula.

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**The strong versions of the order-McShane and Henstock integrals in Riesz space**

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**Abstract**

In this article we consider a strong versions of the order-McShane (Henstock) integral on Banach lattice. We define the property  $S^*oM$  ( $S^*oH$ ) and we compare the order type integrals, showing that a strongly order -type integrals respect a.e. equality for the order- bounded functions. Another interesting difference order integrals is that the strong order-McShane integrability of a function imply that it has the property  $S^*oM$  but this condition cannot be used for the strong order-Henstock integrability and the property  $S^*oH$ .

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**Through Unimodular Matrix on SLE using LaTeX**

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**Abstract**

We deal with the considered results comparing the obtained solutions with the exact ones. Expanding the proposed concepts on generating the system of linear equations through matrix as unimodular or unimodular matrix with compound structures, we give in particular the method of generating SLEs. Our work shows the way for this problem on Determined SLEs. We give the solution for students. In this context, our work gives the method in providing exercises on the topic of SLEs for solutions with integers. For this fact we provide the unimodularity of matrix. We claim that a square matrix is said to be unimodular if it has a determinant value of 1 or -1. The inverse of a unimodular matrix and the product of two unimodular matrices is also unimodular. Those facts are used in our article for determining the solution of system of linear equations (SLEs), because it does not involve fractions at all. In particular, we give the method for generating an SLE via a unimodular matrix with LaTeX. We consider the result and any example of determined SLE with solution in the form of integers. By using LaTeX and python programs, we provide and generalize the way for generating a finite SLEs easily. Some conclusions help to understand the proces.

**Keywords:** unimodular matrix, latex, python, solution, system of linear equations, transformation,

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**Translation Hypersurfaces and Curvatures in the Four Dimensional Euclidean Space**

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**Abstract**

In this study, we consider and examine differential geometry of the translation hypersurfaces in the four dimensional Euclidean space  $\mathbb{E}^4$ . We compute the curvatures  $\mathfrak{C}_i$ , where  $i = 1, 2, 3$ , of the translation hypersurface. In addition, we give some relations for the curvatures of the hypersurface.

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**Unbounded Convergence Structure Properties in Riesz Spaces**

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**Abstract**

In this talk, we study the convergence structure properties with respect to unbounded convergences. We first examine the nets for these convergences. Then we define the unboundedness and lastly, based on these definitions, we discuss order convergence in Riesz spaces.

**Keywords:** Unbounded convergence, order convergence, Riesz spaces

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**Use of Fractional Calculus in Science and Engineering**

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**Abstract**

In this work, a brief history and development of fractional calculus (FC) in the literature is mentioned. Definitions of some fractional derivatives available in the literature are presented. Moreover, the use of FC in various disciplines is presented by giving examples from the literature.

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**Wiener-Hopf Analysis of Sound Propagation in a Duct with a Groove in the Inner Wall**

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**Abstract**

The propagation of plane sound wave in a coaxial pipe with a groove in the inner wall is studied. By applying the Wiener-Hopf method, a Wiener-Hopf equation whose solution involves infinite number of unknown coefficients, which are solved numerically, is obtained. An analytical solution for the field terms are determined. The effect of the groove to reducing unwanted noise is evaluated.

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**$\alpha$ -Admissible multi-valued mappings and related common fixed point theorems**

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**Abstract**

In this paper, new common fixed point theorems are presented. Indeed, we will propose new theorems related to the fixed points of some operators.

We discuss the admissibility of two multi-valued mappings in the category of complete b-metric spaces to obtain the existence of a common fixed point. The obtained results are used for the sake of proving the existence of a positive solution of a coupled system of fractional differential equations.

**Acknowledgement:** Common fixed point; positive solution;  $\alpha$ -Admissible multi-valued mapping.

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